JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 20, No. 3, July–September 2006

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Calculation of Pressure Loads in the Heater of a Hypersonic Blowdown Tunnel

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Nomenclature

A	=	cross-sectional area
C	=	constriction ratio
D	=	diameter
L	=	length
t	=	time
x	=	axial location
z	=	initial diaphragm pressure ratio

Subscripts

orifice	=	conditions at the orifice
1, 2, 3	=	conditions in the number pipe

Introduction

A STRONG rarefaction wave can be created when flow is initiated in a hypersonic blowdown tunnel by the bursting of a diaphragm. This rarefaction can generate significant pressure drops in an upstream component, such as a heater, which is already under severe thermal and structural stresses. Rupture of the heater can be catastrophic, due to the high pressures and temperatures contained within. These pressure drops can be attenuated with the use of a metering orifice, which requires an accurate prediction of the pressure drop for proper sizing. This pressure drop could be determined computationally, but to be generally applicable and to provide physical insight, a closed-form or simple numerical solution is of value. Three methods of doing so are investigated: acoustic reflection, flow pattern assumption, and the classical method of characteristics.

The acoustic method is based on transient propagation theory, designed for handling pressure surge in pipes. The assumption of incompressibility allows superposition of incident and reflected waves, leading to simple algebraic solutions of the continuity equations. Although compressibility is critical in the analysis of the downstream, hypersonic flow, the Mach number in the reservoir is subsonic, and usually well below the compressibility limit of 0.3. Acoustic theory considers a wide variety of boundaries. In this Note, only conditions likely to be seen in a blowdown tunnel will be considered: junctions between two pipes of different areas, junctions of three pipes, and dead ends. Treatment of these cases, as well as many others is discussed in Refs. 1 and 2.

The flow pattern method is based on a family of methods used to treat flow through a single area change, such as in the diaphragm region of a shock tube. Multiple area changes can be treated in sequence by identifying the flow pattern at each change and applying the correct relations.[3] This method is similar in nature to the acoustic one, except that waves are treated as shocks or expansions, rather than perturbations. The result is a system of nonlinear equations, which must be solved numerically, relaxing the assumption of incompressibility at the cost of extra computation time. These systems of equations are derived and explained in detail in the literature: Analysis of an area change within the diaphragm section was analyzed by Alpher and White[4]; the patterns generated by a rarefaction moving through an area constriction or expansion were investigated by Gottlieb and Igra[5] and Igra and Gottlieb.[6]

Finally, both methods are compared to the one-dimensional unsteady method of characteristics. This classical method provides a time-dependent solution to the inviscid Navier–Stokes equations. Unlike the preceding two methods, which are quasi steady, this method readily handles nonlinear wave interactions. Such an analysis is frequently used when considering the compressible flow through a complex network of pipes and is well documented in texts such as Ref. 7.

These three methods vary systematically in the number of simplifying assumptions made and in the difficulty of calculating the solution. By the examination of the three methods in conjunction, tradeoffs between complexity and physical accuracy can be analyzed.

Model Tunnel

To validate the chosen models, a generic model heater has been defined, representative of existing facilities. The model heater (Fig. 1) contains a variety of area changes, all of which might be encountered in the geometry of a real heater.

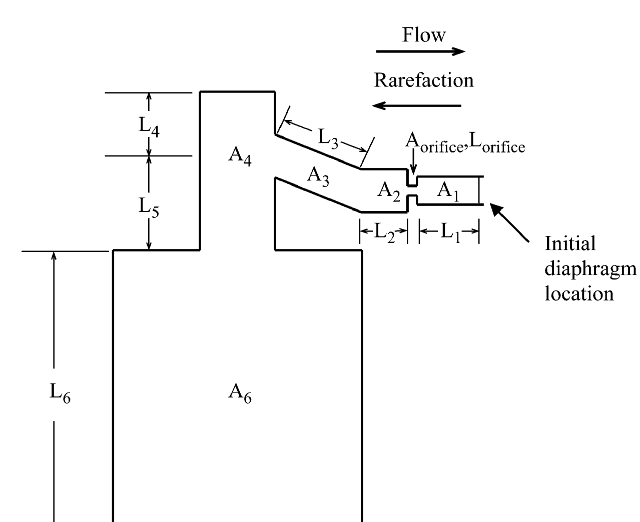

Fig. 1 Schematic of model tunnel.

Received 19 July 2005; revision received 15 August 2005; accepted for publication 17 August 2005.

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In the model, as with the standard convention, flow moves from left to right, and the rarefaction wave propagates from right to left. The diaphragms initially separate a high-pressure region on the left from a low-pressure region on the right. A wave is generated as the diaphragm is burst and immediately begins to travel a distance L_1 through a horizontal pipe of area A_1. The next region is a metering orifice of length L_{orifice}: a plate with a small effective area, used to reduce the pressure drop to the heater by choking the flow. This effective area may be accomplished by a single hole in the middle of the plate, or an array of holes. It is assumed that a quantity, effective area A_{orifice}, can be generated, which performs similarly to an orifice with a single hole of that area. Upstream of the orifice, the diameter of the tunnel increases to A_2 for a distance of L_2, where $A_2 > A_1$. Many tunnels contain such an area change because it has been shown to increase the strength of the generated shock for a given initial pressure ratio.[1] Next, an angled pipe, area A_3 and length L_3, is included because the reservoir may not be located inline with the rest of the tunnel. L_3 is taken parallel to the axis of the pipe, not horizontally. Because the analyses presented here are one dimensional, the angle itself does not affect the flow. Instead, the angled pipe represents the need for an area reduction to accommodate the mitered joint. This pipe enters the top of the heater, a vertical pipe of area A_4. A length L_4 of the heater lies above the angled pipe, and L_5 lies below. The pipe below the angled pipe junction then connects to the largest portion of the reservoir with area A_6 and length L_6.

All cross sections are assumed to be circular, of diameters $D_1 = 0.25$, $D_2 = 0.275$, $D_3 = 0.271$, $D_4 = 0.375$, and $D_5 = 1.25$ respectively. The lengths are $L_1 = 1$, $L_2 = 0.5$, $L_3 = 2$, $L_4 = 0.5$, $L_5 = 1$, $L_6 = 2$, and $L_{\text{orifice}} = 0.1$. All units are in meters. Orifice size shall be treated as a variable so that behavior can be qualified as a function of orifice geometry. A new non-dimensional parameter, constriction ratio C, shall be introduced, where

$$C \equiv A_{\text{orifice}}/A_1 \tag{1}$$

The other independent variable is the initial diaphragm pressure ratio: the ratio of the high-pressure initially upstream of the diaphragms (which will also be the stagnation pressure) to the low-pressure downstream of the diaphragms, designated z.

Results

Comparing the acoustic to the flow pattern method is very straightforward because both methods generate a single value of pressure change for a given orifice size. The method of characteristics (MOC), however, provides an entire function of t and x for each value of C. Figure 2 shows pressure traces in each pipe for $C = 40\%$. Pressure traces are taken at three axial locations for each pipe: at the entry and exit boundaries and at the center. Predictions for incident and reflected pressures from the quasi-steady methods are plotted as well.

For the horizontal pipe and the top and bottom sections of the heater, pressure is essentially uniform across the entire length of the pipe at any given time. For the angled pipe, the pressure exhibits the same pattern of oscillations across the pipe, but varies significantly in magnitude. In the midsection of the heater, pressure varies in magnitude and character across the length of the pipe. The horizontal pipe and the ends of the heater are each only "communicating" with one other pipe, with a static boundary condition on the other end. Pressures in the angled pipe and the middle of the heater are being

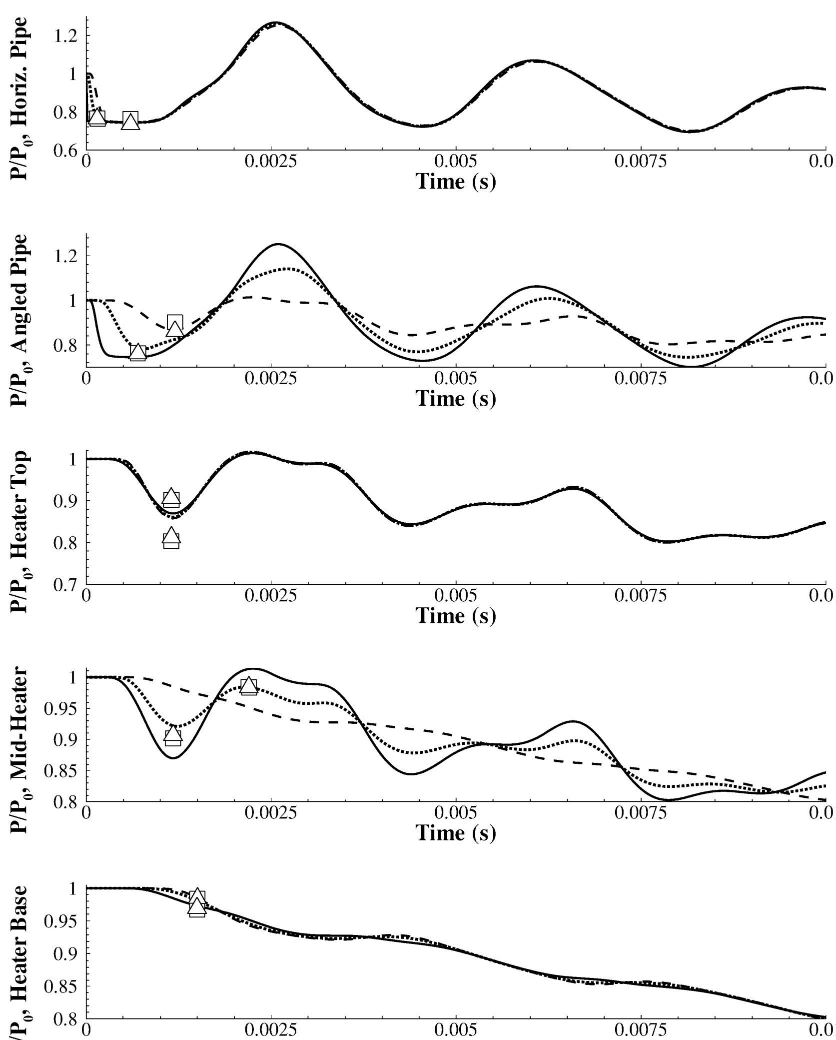

Fig. 2 Pressure traces for $C = 40\%$: ——, MOC entry; · · · ·, MOC middle, – – –, MOC exit; □, acoustic; and △, flow pattern.

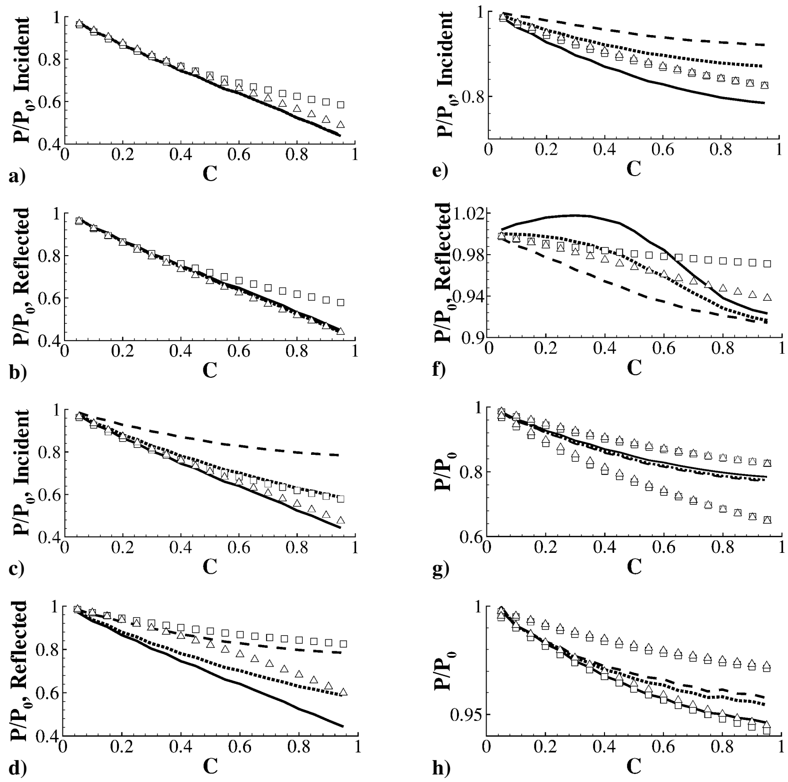

Fig. 3 Comparison of pressure predictions between methods: a) incident waves, horizontal; b) reflected waves, horizontal; c) incident waves, angled pipe; d) reflected waves, angled pipe; e) incident waves, middle of heater; f) reflected waves, middle of heater; g) top of heater; and h) bottom of heater: ——, MOC entry; · · · ·, MOC middle; – – –, MOC exit; □, acoustic; and △, flow pattern.

driven by varying states at both ends, which leads to significant variations in pressure with x.

These pressure variations over x and t make it difficult to compare the quasi-steady predictions with the MOC. In some of the traces, distinct minima are present providing obvious comparison values, but not always. In the horizontal pipe the incident wave is a rarefaction. The junction with the angled pipe is an area contraction, and so the reflected wave is also a rarefaction. Therefore, the incident wave will not form a minimum, but at best, a plateau, and at worst, a line that passes through that point. For the case of the horizontal pipe, a plateau will be seen in the pressure trace at the downstream boundary, but at any upstream value of x, it will be absorbed into the reflected rarefaction. For the angled pipe, the incident wave is a rarefaction, and the reflected wave is a compression. The incident wave will form a distinct minimum, and theoretically, the reflected wave should generate a maximum. This is not the case. Instead, the reflected wave forms a small plateau (if at all, at the upstream boundary, the incident and reflected waves merge together into a single minimum) then rises to an overpressure not predicted by either quasi-steady method. Upstream of the angled pipe, the transmitted rarefaction through the heater middle reflects off the area change in the heater. As it travels downstream, it is intensified by the junction with the angled pipe. (Because the wave is now traveling downstream, it sees this junction as an area reduction.) One could generate further acoustic predictions on this phenomenon, but without adding some sort of time dependence, it would be difficult to determine which waves should be summed. A time-dependent acoustic method would not be difficult to implement computationally, but if so much computational effort were acceptable, the MOC would be a better choice.

For the heater bottom, the incident and reflected waves are so close together that the graph more closely resembles a linear decrease in pressure than an oscillation. At the downstream junction, plateaus can be distinguished, but for all practical purposes, this prediction would have little value in determining loads on structural members. For this case, it is fortunate that pressure changes in the bottom of the heater are very small and gradual in comparison to changes in pipes that are farther upstream. The bottom portion of the heater can be designed safely to the specifications of the rest of the apparatus.

Figures 3a–3h show a comparison of all three methods, where the values for MOC are taken as the appropriate maximum, minimum, or plateau, as discussed. For the top of the heater, there is only a single relevant pressure drop, which is compared to both the incident and reflected predictions from the quasi-steady methods. For the bottom of the heater, as discussed earlier, there are no significant minima, although plateaus can be distinguished near the entry if C is very small. Instead of taking actual plateaus, the quasi-steady methods are compared to characteristics solutions at the time value for which this plateau is located (for this example, $t = 0.0017$ s). From the locations of the various minima in other pipes, it can be seen that the locations of the various flow structures vary only slightly in time with different values of C. The upstream boundary of the middle pipe is found in a similar manner.

In general, it has been found that the quasi-steady methods are far more accurate for small values of C. Furthermore, when C is small, there is little difference between the flow pattern method and the acoustic method. The acoustic method is far easier to implement and can be iterated if one wished to attempt to predict the reflected overpressures However, at least in the case of the horizontal pipe, the flow pattern method gave good matching to higher values of C than the acoustic method. In general, the quasi-steady methods did not appear to correspond to any particular axial location, although in comparison to experiment, one might expect that the approximations would be less accurate in the vicinity of the stepped area change section because separation at the corners of the area change would decrease the effective flow area. Although it is easy to match the quasi-steady results to the MOC results, the quasi-steady results by themselves may or may not be useful in the ultimate goal of predicting the stress loading on the structural members.

Summary

The purpose of this study has been to investigate three possible methods of predicting the transient pressure behavior in the heater of a hypersonic-blowdown tunnel caused by the starting. One method, the flow pattern assumption method, was introduced in this study using a combination of existing methods that had not previously been used in conjunction. In addition, comparing the results of methods based on different assumptions has granted some insight on the important features of the flow.

For small values of constriction ratio, all three methods generate similar solutions. As C increases, the acoustic method loses accuracy, although the other two methods provide similar results. However, the two quasi-steady methods are unable to predict time-dependent effects, such as wave interactions, as well as incorporating the temporal behavior of variable-area orifices or vented liners. Although it is easy to match the quasi-steady results to the MOC results, the quasi-steady results by themselves may or may not be useful in the ultimate goal of predicting the stress loading on the structural members.

The quasi-steady methods generate acceptable approximations for the first pressure drop as long as C is less than about 40–50%, but gives no further characterization of the pressure behavior. Although the MOC is computationally more intensive, it provides significantly more insight.

Acknowledgments

Portions of this work have been supported by Arnold Engineering Development Center, with John Lafferty as Technical Monitor and by the Space Vehicle Technology Institute under Grant NCC3-989 jointly funded by NASA and the Department of Defense within the NASA Constellation University Institutes Project, with Claudia Meyer as the Project Manager.

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